Hexagonal Picture Languages Generated By Assembling Hexagonal Tiles

Dr.Anitha P
Associate Professor of Mathematics
BJM Government College, Chavara

We propose a new formalism for generating hexagonal picture languages based on assembling of hexagonal tiles and hexagonal dominos that uses rules having two sites namely context site and a replacement site. More briefly a hexagonal picture can be generated from a finite set of initial hexagonal picture by iteratively applying the rules from a given set of rule sequences called a Hexagonal Tiling Rule System (HRTS). We claim that this HRTS system have a greater generative capacity than Hexagonal Tiling System (HTS), even in the case of one letter alphabet. This is possible due to the repeated use of replacement site.

Keywords: Hexagonal Tiling Rule System, Hexagonal Rule sequence, Context and replacement sites, Pseudo canonical hexagonal picture, NW rule sequence, SW rule sequence, E rule sequence, derivation picture, computational power.

1. INTRODUCTION

Recently searching for a new method for defining hexagonal pictures has moved towards the new definition for recognizable languages generated by hexagonal pictures which inherits many properties from existing cases, in Restivo and Rozenberg [1997],Restivo and Rozenberg [1996]. Local and recognizable hexagonal picture languages in terms of hexagonal tiling system were introduced and studied in K. S. Dersanambika and Subramanian [2005]. Subsequently hexagonal hv-local picture languages via hexagonal domino systems were introduced in Latteurx and Simplot [1997]. Hexagonal arrays and hexagonal patterns are found in picture processing and image analysis H. Geetha and Kalyani [2011]. Kolam arrays were introduced by Siromoney and Siromoney Siromoney and Siromoney [1976]. In Paola Bonizzoni and Mauri [2009] Paola Bonizzonia et.al defined a formalism for generating picture languages based on assembly mechanism of tiles that uses some specified rules called tiling rule systems.

Based on the operations defined on the hexagonal arrays we defined hexagonal tiling rule system. Hexagonal tiling rule system is a new method for defining hexagonal picture languages that is based on some rules to assemble tiles. In this paper we investigate on hexagonal pictures. More precisely our approach for generating hexagonal pictures is based on the notion of hexagonal tiling rule system on a finite number of hexagonal pictures and a rule that is applied to generate hexagonal picture languages. As in two dimensional cases here also a rule consists of a pair of hexagonal tiles say a context site and replacement site. Context site gives where the rule to be applied and replacement site is used to change part of context site.

In this rule system the rules are simultaneously applied to a hexagonal picture resulting a new hexagonal picture according to hexagonal tiling rule system. A hexagonal tiling rule system is a quadruple (P, R, Σ, Δ) where P is an intial finite set of hexagonal pictures, R is a finite set of hexagonal tiling rules to be applied to hexagonal pictures, Σ , a finite set of alphabets and Δ, a special symbol for border. Compared to hexagonal tiling system, hexagonal tiling rule system has more generative capacity. In this paper preliminaries are discussed as section 2 and in section 3 hexagonal tiling rule systems is introduced. Then the last section deals with the investigation of its computational power and comparison to hexagonal picture languages.

Author addresses:Dr. Anitha.P, anitabenson321@gmail.com,Associate Professor of Mathematics Department of Mathematics, BJM Government College, Chavara

2. PRELIMINARIES

In this section we review the notions of formal language theory and some of the basic concepts on hexagonal pictures and hexagonal picture languages ?].

Let Σ be a finite alphabet of symbols. A hexagonal picture p over Σ is a hexagonal array of symbols of Σ. The set of all hexagonal arrays of the alphabet Σ is denoted by Σ^{**H}. A hexagonal picture over the alphabet a, b, c d is shown in the figure given below

$a \quad a \quad b$
$a \quad b \quad f \quad c$
$b \quad c \quad c \quad a \quad c$
$d \quad a \quad a \quad d$
$c \quad c \quad a$

Figure. 1

With respect to a triad of triangular axes (x , y , z) the co-ordinates of each element of the hexagonal picture in Figure 2(a) and Figure 2(b) respectively are given below K. S. Dersanambika and Subramanian [2004].

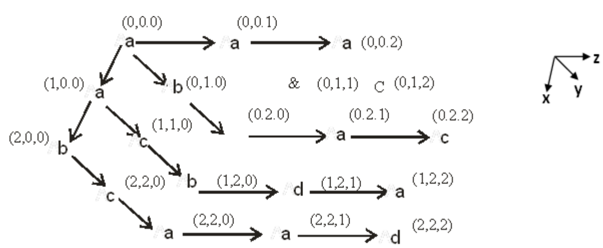


Figure. 2a

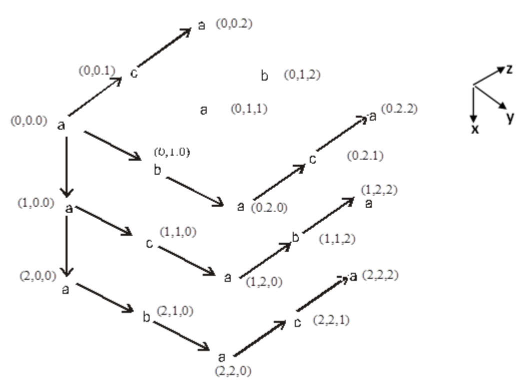


Figure. 2b

If $p \in \Sigma^{**H}$, then $\hat{p}$ is the hexagonal picture obtained by surrounding p with a special boundary # is called a bordered hexagonal picture which is shown in Figure 3.

Let $l_1(p) = l, l_2(p) = m, l_3(p) = n$ be the size of the hexagonal arrays. We write p = (l, m, n), the size of a picture. For a picture p of size (l, m, n) we have the bordered picture $\hat{p}$ is of size (l+1, m+1, n+1).

```
      #   #   #   #
    #   a   b   c   #
  #   b   b   a   b   #
#   c   a   b   c   f   #
  #   d   b   e   d   #
    #   c   f   a   #
      #   #   #   #
```

Figure. 3

Now we see the projections of hexagonal picture and projections of a language. I and Σ be two finite alphabets and $\Pi : \Gamma \rightarrow \Sigma$ be a mapping, this mapping π is called a projection.
A hexagonal tile is of the form as shown in Figure 4.

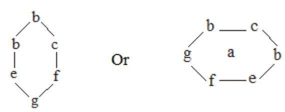


Figure. 4

Given a hexagonal picture p of size (l , m , n) we denote the set of hexagonal subpicture of p of size (2, 2, 2) is called a hexagonal tile of size (2, 2, 2). Figure 4 denote a hexagonal tile of size (2, 2, 2).
A hexagonal tiling system [**?**] T is a 4-tuple $(\Sigma, \Gamma, \pi, \theta)$ where Σ and Γ are two finite set of symbols. $\pi : \Gamma \longrightarrow \Sigma$ is a projection and θ is the set of hexagonal tiles over the alphabet $\Gamma \cup \{\#\}$. A hexagonal picture has got three types of dominos as shown in Figure 5.

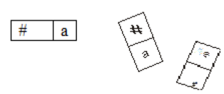


Figure. 5

Definition 1

Let p be a hexagonal picture of size (l, m, n), a partial bordered hexagonal picture $\hat{p}$ is a hexagonal picture of size (l, m + 1, n + 1) or (l + 1, m, n + 1) or (l + 1, m + 1, n). A picture p can be obtained from $\hat{p}$ by adding borders partially along the NW, NE, SW, SE, E and W arrowhead directions respectively.
Definition 2

A pseudo - canonical hexagonal picture is a hexagonal picture same as bordered hexagonal picture as described in Figure 3. In this paper we use two types of hexagonal pictures, 1) pseudo - canonical hexagonal picture and 2) partially bordered hexagonal pictures as defined above.
Example 1
Partially bordered hexagonal pictures is shown in Figure 6.
Definition 3

A hexagonal sub picture $\hat{p'}$ is a picture which is a hexagonal sub array of the picture $\hat{p}$. Given a hexagonal picture $\hat{p}$ then $B_{l,m,n}(\hat{p})$ denotes the set of hexagonal sub pictures of size l, m, n.

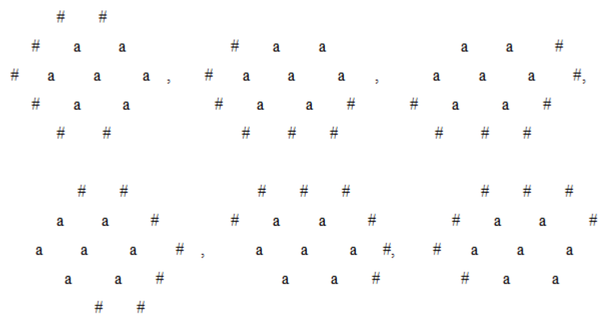


Figure. 6

3. HEXAGONAL TILING RULE SYSTEM

In this section we define the notion of hexagonal tiling rule and hexagonal tiling rule systems. A general tiling rule is defined over a pair t_1, t_2 of tiles in θ where t_1 is a context site rule of r and t_2 is replacement site rule. Then r is denoted by $r : t_1 \longrightarrow t_2$. In the case of hexagonal pictures we distinguish three types of rules: NW arrowhead rule, SW arrowhead rule and E arrowhead rule. The context site of NW arrowhead rule, SW arrowhead rule and E arrowhead rule are denoted respectively by Figure 7.

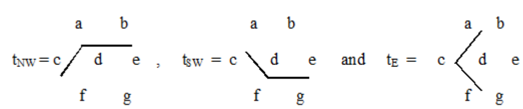


Figure. 7

When the rule r = t_{NW} is applied t_{NW} replaces the NW arrowhead of the hexagonal tile and rule r = t_{SW} is applied t_{SW} replaces the SW arrowhead of the hexagonal tile while rule r = t_E is applied t_E replaces the E arrowhead of the hexagonal tile. The three rules acts together to enlarge the hexagonal picture. This fact can be used to formalize the NW arrowhead rule sequence, SW arrowhead rule sequence and E arrowhead rule sequence. A similar rule can be established if we use SE arrowhead for NW arrowhead, NE arrowhead for SW arrowhead and W arrowhead for E arrowhead as these represent the bottom and top arrowheads in the same direction. Hence here we consider only the above mentioned three rules. The combined use of the three rules results in Figure 8 shown below.

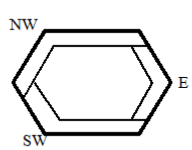


Figure. 8

Definition 4 (NW rule sequence)

A sequence $S = NW_1, NW_2, ..., NW_m$ of rules is a NW rule sequence in short NW sequence if and only if for each $1 \leq j \leq m$ it holds that

The application of rules in S defines the pseudo - canonical pictures $p_{(S,NW)}$ and $q_{(S,NW)}$ of size (2, m + 1, 2) called the context site and replacement site respectively such that $p_{(S,NW)}(0, j, 0) = a, p_{(S,NW)}(1, j, 0) = c, p_{(S,NW)}(0, j, 1) = b, q_{(S,NW)}(0, j, 0) = i, q_{(S,NW)}(1, j, 0) = h, q_{(S,NW)}(0, j, 1) =$

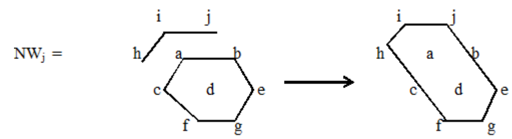


Figure. 9

j, for each North west arrowhead.

Definition 5 (SW rule sequence)

A sequence $S = SW_1, SW_2, ..., SW_m$ of rules is a SW rule sequence in short SW sequence if and only if for each $1 \leq i \leq l$ it holds that

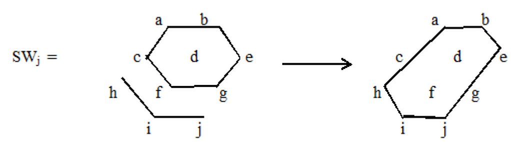


Figure. 10

Given a SW- rule sequence as in the above definition the application of S produces the pseudo - canonical pictures $p_{(S,SW)}$ and $q_{(S,SW)}$ of size (l +1, 2, 2) called the context site and replacement site respectively such that $p_{(S,SW)}(l, 1, 0) = c, p_{(S,SW)}(l, 2, 0) = f, p_{(S,SW)}(l, 2, 1) = j$, while $q_{(S,SW)}(l, 1, 0) = h, q_{(S,SW)}(l, 2, 0) = i, q_{(S,SW)}(l, 2, 1) = j$, for each for each South west arrowhead. *Definition 6* (E rule sequence)

A sequence $S = E_1, E_2, ..., E_k$ of rules is a E rule sequence in short E sequence if and only if for each $i \leq k \leq n$ it holds that

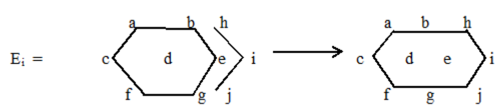


Figure. 11

The application of rules in S defines the pseudo - canonical pictures $p_{(S,E)}$ and $q_{(S,E)}$ of size (2, 2, k+1) called the context site and replacement site respectively such that $p_{(S,E)}(0, 1, k) = b, p_{(S,E)}(0, 2, k) = e, p_{(S,E)}(1, 2, k) = g$, while $q_{(S,E)}(0, 1, k) = h, q_{(S,E}(0, 2, k) = i, q_{(S,E)}(1, 2, k) = j$, for each for each East arrowhead.
Example 2
Consider Figure 12a. It gives the the NW rules.
Then S $=NW_1NW_2NW_3$ is a NW sequence picture then
Example 3
Consider Figure 13a. It gives the the SW rules.
Then S $=SW_1SW_2SW_3$ is a SW sequence picture then *Example 4*
Consider Figure 14a. It gives the the E rules. be the E rules. Then S $= E_1E_2E_3$ is a E sequence picture then

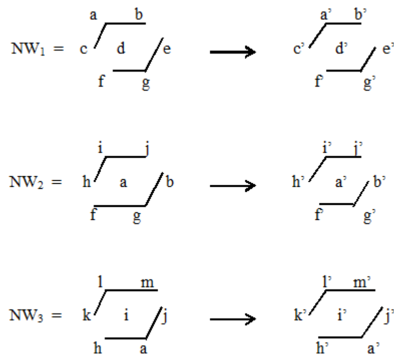


Figure. 12a

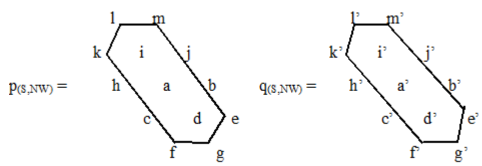


Figure. 12

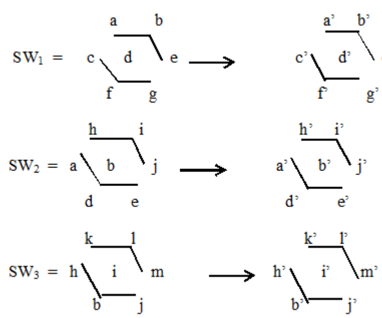


Figure. 13a

Definition 7 (Application of NW rule sequence)

Let S = $NW_1 NW_2 ... NW_{m+1}$ be a sequence of NW rule having context site $p_{(S,r)}$ and replacement site $q^{NW}_{(S,r)}(1) = [a^{'}_1, ..., a^{'}_{m+2}], q^{NW}_{(S,r)}(2) = [b^{'}_1, ..., b^{'}_{m+2}]$. We can apply S to the picture $\hat{p}$ obtained from p or p^{-} having sizes (l, m, n), (l + 1, m + 1, n) respectively. We apply the rule to the i^{th} NW arrowhead position in a hexagonal picture where $1 \leq i \leq m$ and if and only if the following holds.

- $\hat{p}^{NW}[i, ..., i+1] = p_{(S,NW)}$ and
- $[a^{'}_1, ..., a^{'}_{m+2}]$ is over Σ for NW arrowhead i with $1 \leq i \leq m$ while $[b^{'}_1, ..., b^{'}_{m+2}]$ is over Σ with $1 \leq i \leq m-1$.

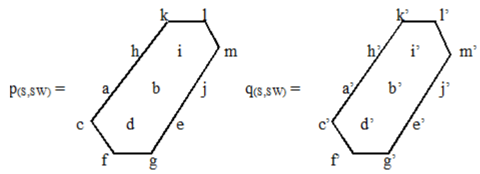


Figure. 13

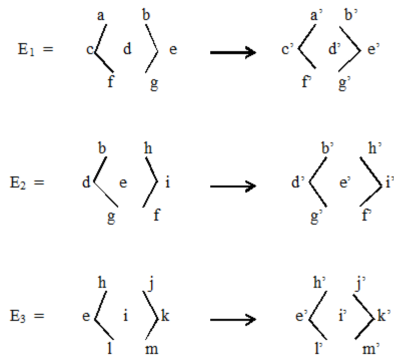


Figure. 14a

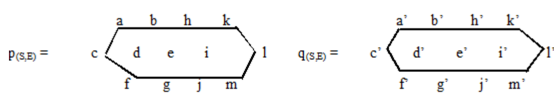


Figure. 14

Definiton 8 (Application of SW rule sequence)

Let S = $SW_1SW_2...SW_{m+1}$ be a sequence of SW rule having context site $p_{(S,r)}$ and replacement site $q^{SW}_{(S,r)}(1) = [a^{'}_1, ..., a^{'}_{m+2}], q^{SW}_{(S,r)}(2) = [b^{'}_1, ..., b^{'}_{m+2}]$. We can apply S to the picture $\hat{p}$ obtained from p or p^{-} having sizes (l, m, n), (l , m + 1, n + 1) respectively. We apply the rule to the i^{th} SW arrowhead position in a hexagonal picture where $1 \leq i \leq l$ and if and only if the following holds.

- $\hat{p}^{SW}[i, ..., i+1] = p_{(S,SW)}$ and
- $[a^{'}_1, ..., a^{'}_{m+2}]$ is over Σ for SW arrowhead i with $1 \leq i \leq l$ while $[b^{'}_1, ..., b^{'}_{m+2}]$ is over Σ with $1 \leq i \leq l-1$.

Definition 9 (Application of E rule sequence)

Let S = $E_1E_2...E_{m+1}$ be a sequence of E rule having context site $p_{(S,r)}$ and replacement site $q^{E}_{(S,r)}(1) = [a^{'}_1, ..., a^{'}_{m+2}], q^{E}_{(S,r)}(2) = [b^{'}_1, ..., b^{'}_{m+2}]$. We can apply S to the picture $\hat{p}$ obtained from p or p^{-} having sizes (l, m, n), (l + 1, m, n + 1) respectively. We apply the rule to the i^{th} E arrowhead position in a hexagonal picture where $1 \leq i \leq n$ and if and only if the following holds.

- $\hat{p}^{E}[i, ..., i+1] = p_{(S,E)}$ and
- $[a^{'}_1, ..., a^{'}_{m+2}]$ is over Σ for E arrowhead i with $1 \leq i \leq n$ while $[b^{'}_1, ..., b^{'}_{m+2}]$ is over Σ with $1 \leq i \leq n-1$.

Observe that condition 2 in the above three definitions is given to exclude the insertion on border inside a hexagonal picture. The effect of this application is shown in the Figure 15.

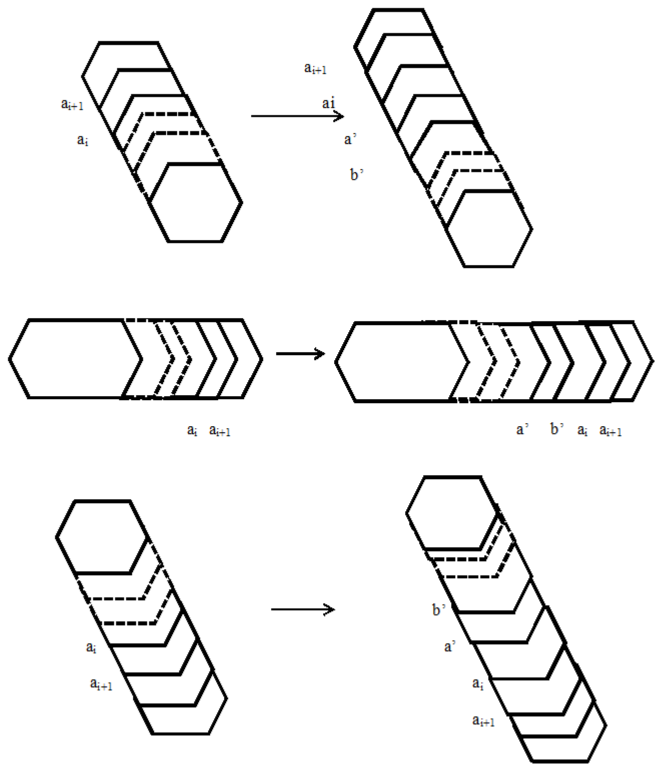


Figure. 15

Definition 10 (NW arrowhead derived picture)

Let S be a NW-sequence such that S can be applied at the i^{th} position of NW-arrowhead of a hexagonal picture $\hat{p}$ of size (l', m', n') obtained from the hexagonal picture p and p^- of sizes (l, m, n) and (l + 1, m + 1, n) respectively. Then the hexagonal picture $\hat{q}$ derived from $\hat{p}$ by applying S such that $\hat{q}^{NW}[1, ..., i] = \hat{p}^{NW}[1, ..., i].\hat{q}^{NW}[i+1, ., i+2]$ is equal to the replacement site q(S,NW) by applying S and $\hat{q}^{NW}[i+3, ..., m'+1] = \hat{p}^{NW}[i+2, ..., m']$.
Definition 11 (SW arrowhead derived picture)

Let S be a SW-sequence such that S can be applied at the i^{th} position of NW-arrowhead of a hexagonal picture $\hat{p}$ of size (l', m', n') obtained from the hexagonal picture p and p^- of sizes (l, m, n) and (l , m + 1, n + 1) respectively. Then the hexagonal picture $\hat{q}$ derived from $\hat{p}$ by applying S such that $\hat{q}^{SW}[1, ..., i] = \hat{p}^{SW}[1, ..., i].\hat{q}^{SW}[i+1, ., i+2]$ is equal to the replacement site q(S,SW) by applying S and $\hat{q}^{SW}[i+3, ..., l'+1] = \hat{p}^{SW}[i+2, ..., l']$.
Definition 12 (E arrowhead derived picture)

Let S be a E-sequence such that S can be applied at the i^{th} position of E-arrowhead of a hexagonal picture $\hat{p}$ of size (l', m', n') obtained from the hexagonal picture p and p^- of sizes (l, m, n) and (l + 1, m, n + 1) respectively. Then the hexagonal picture $\hat{q}$ derived from $\hat{p}$ by applying S such that $\hat{q}^E[1, ..., i] = \hat{p}^E[1, ..., i].\hat{q}^E[i+1, ., i+2]$ is equal to the replacement site q(S,E) by applying S and $\hat{q}^E[i+3, ..., n'+1] = \hat{p}^E[i+2, ..., n']$.

The application of S to $\hat{p}$ to derive $\hat{q}$ is denoted by $\hat{p} \longrightarrow S\hat{q}$. Also the iterated application of S over a hexagonal picture $\hat{p}$ to generate hexagonal picture $\hat{q}$ is denoted by $\hat{p} \longrightarrow^i S\hat{q}$. A hexagonal picture $\hat{p}'$ derived from a hexagonal picture $\hat{p}$ by applying these NW or SW or E rule sequences is denoted by $\hat{p} \Longrightarrow R\hat{p}'$ if and only if there exist a rule sequence $S_1, ..., S_k$ such that $\hat{p} \longrightarrow S_1\hat{p_1} \longrightarrow S_2\hat{p_2}... \longrightarrow S_k\hat{p}'$ which is called the derivation of $\hat{p}'$ from $\hat{p}$ while d = $S_1, ..., S_k$ is called derived sequence applied for getting $\hat{p}'$ from $\hat{p}$.

A derivation sequence for hexagonal pictures d = $S_1, ..., S_k$ is unambiguous if and only if for each i $1 \leq i < k$, the context site of S_{i+1} is the replacement site S_i. A derivation sequence is deterministic if there exist a deterministic way in the occurrence of replacement site rules. In such cases S_i can be followed only by S_{i+1} .

Given an initial finite set of hexagonal pictures and a finite set of rules, the rules can be combined to produce NW-sequence, SW- sequence or E sequence. Again the rules can be applied iteratively to the initial picture to generate infinite languages of hexagonal pictures. This process of generating hexagonal pictures is described by the notion of a hexagonal tiling rule system and language generated by such type of systems are called L(HTRS).

Definition 13 (Hexagonal Tiling Rule System)

A hexagonal tiling rule system, in short HTRS is a quadruple T = $(P, R, \Sigma, \#)$, where P is the finite set of hexagonal pictures, R is the set of rules say NW, SW, E sequence rules, Σ is the finite set of alphabets and # the symbol disjoint from Σ, the finite set of alphabets used for representing borders.

Definition 14

Consider a HTRS system T, then the language generated by L(HTRS) is a set $\{p : \hat{p} \in L\}$ where L = $P \cup \{\hat{p_1} : \hat{p} \Longrightarrow R\hat{p_1}, p \in P, \hat{p_1}$ is cannonical $\}$. The language L is called the hexagonal canonical language generated by the HTRS system.

Remark 1

Assume that $p' \in L(T)$,the derivation sequence applied on it, say $\hat{p} \longrightarrow S_1\hat{p_1} \longrightarrow S_2\hat{p_2}... \longrightarrow S_{k-1}\hat{p_{k-1}} \longrightarrow S_k\hat{p}'$, then $\hat{p} \in P$ The intermediate pictures $\hat{p_i}$ with $1 \leq i < k$ are not necessary canonical pictures but may be pseudo - canonical ones.

4. COMPUTATIONAL POWER OF HTRS SYSTEMS

In this section we investigate the computational power of HTRS system. Now the class of hexagonal picture languages generated by HTRS system includes the one of recognizable hexagonal languages. We first show that hexagonal recognizable picture languages are generated by HTRS systems. Then we show that the generative capacity of Hexagonal tiling rule system is greater than Hexagonal tiling system.

Theorem 1

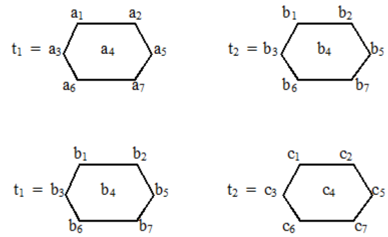
L(HTS) $\subseteq$ L(HTRS).

Proof

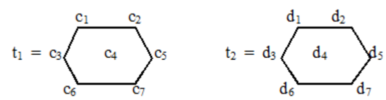
Let L be a recognizable hexagonal picture language and let T = $(\Sigma, \Gamma, \pi, \theta)$ be a hexagonal tiling system where Σ and Γ are two finite set of symbols,$\pi : \Gamma \longrightarrow \Sigma$ is a projection and θ is a finite set of hexagonal tiles over the alphabet $\Gamma \cup \{\#\}$. Let us define the following operations $=_{NW}, =_{SW}$ *and* $=_E$ over the tiles in θ : $t_1 =_{NW} t_2$ if and only if

where $a_2 = b_5, a_4 = b_7, a_3 = b_6, a_1 = b_4$.

$t_1 =_{SW} t_2$ if and only if



where $b_3 = c_1, b_4 = c_2, b_7 = c_5, b_6 = c_4$.
$t_1 =_E t_2$ if and only if



where $c_2 = d_1, c_4 = d_3, c_5 = d_4, c_7 = d_6$.
In the following we define a HTRS system T $= (P, R, \Sigma, A)$ for generating the language L where P is a finite set of hexagonal pictures consisting of the empty picture

$$\begin{array}{ccccc} & \# & & \# & \\ \# & & \# & & \# \\ & \# & & \# & \end{array}$$

and A contains the finite set of alphabet $\Sigma \cup \Gamma \cup A'$ where A'=$\{(a, b, c) : a, b, c \in \Gamma\}$.

R be the set of rules listed below and grouped according to the pair of tiles in θ, by the relations $=_{NW}, =_{SW}$ *and* $=_E$ respectively. Applying the rules we should get the tiling of a local hexagonal picture and at the same time the projection of the hexagonal local language. In order to do so given a pair of hexagonal tiles t_1 and t_2 such that $t_1 =_{NW} t_2$ we built a north west arrowhead rule having a replacement site given by a tile t such that the top NW- arrowhead of t will project the upper NW domino of the tile t_2 while the bottom NW- arrowhead of t_2 memorizes by using the symbols in A, the tile t_2 that will have the NW- arrowhead projected. Similarly for a context site rule NW that is given a pair of hexagonal tiles where
then we built a new NW rule of the form
Similarly we can built rules for pair of hexagonal tiles related by $=_{SW}$ and $=_E$ relation. The rules R are listed below in Figure 16.

The rules R_{NW1} and R_{NW2} can be used on top NW- arrowhead and bottom NW arrowhead respectively. Similarly the other two sets can be used for SW arrowhead and E arrowhead. Now p $\in$ L be a hexagonal picture of size (l, m, n) that is the projection of the local picture q. Then we show that the hexagonal picture is generated by the above rules repeatedly. Assume that l = m = n = 2. Then the rule R_{NW1} is used to a hexagonal picture shown in Figure 17 to generate a hexagonal picture $\hat{q}[1, 2]$. The above picture can be produced to a canonical picture by applying the rules $R_{SW1}$and R_{E1}repeatedly.The resultant picture is shown in Figure 18.
Case : 1
Assume that l = 2, m > 2, n > 2. For convenience we take l = 2, m = 3, n = 3. First we apply E_1 rule to the Figure 17, then applying NW_2 rule to the bottom arrowhead we could generate $\hat{q}[2, 3, 3]$. See Figure 19.
Case:2
Assume that l > 2, m = 2, n > 2. Here we take l = 3, m = 2, n = 3. First we apply SW_1 rule to the Figure 17, then applying E_2 rule we could generate $\hat{q}[3, 2, 3]$.See Figure 20.

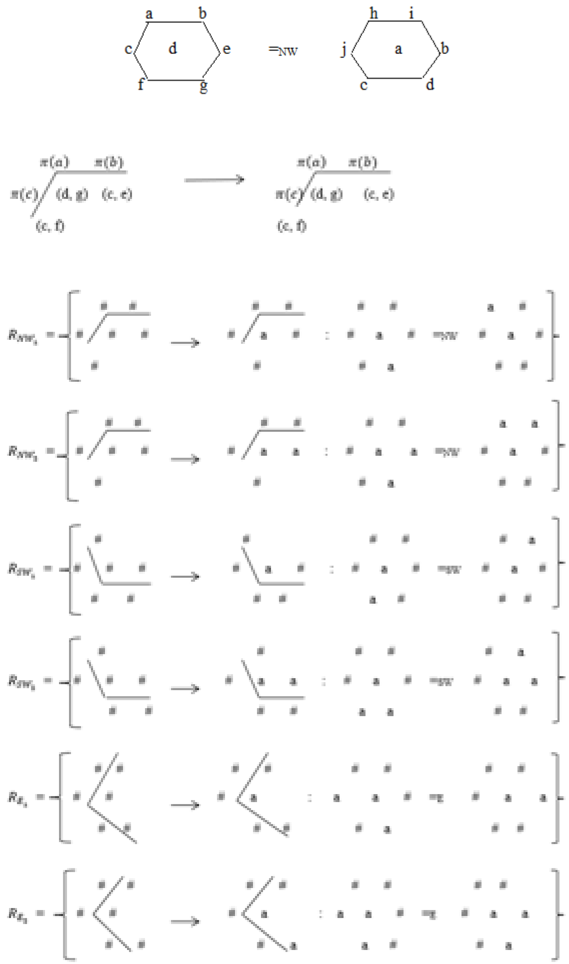


Figure. 16

Case:3

Assume that l > 2, m > 2, n = 2. As usual here also we take l = 3, m = 3, n = 2. First we apply SW_1 rule to the Figure 17, then applying NW_2 rule we could generate $\hat{q}[3, 3, 2]$.See Figure 21.

In all the above three cases we could extend the hexagonal picture to higher dimensions by successfully applying the rules. A regular hexagonal picture can be obtained by applying all the rules simultaneously as shown in the Figure 18.

Now let us show that L(HTS) $\subseteq$ L(HTRS). For that it is enough to show that a picture $\hat{q} \in$ L(HTS) there exist a picture $\hat{p}$ such that $B_{2,2,2}(\hat{p}) \subseteq \theta \wedge \hat{q} = \pi(\hat{p})$,where π is the projection mapping. The proof can be explained to a hexagonal picture of size (l, m, n) where l, m, n $\geq$

\# \#
\# a \#
\# \#

Figure. 17

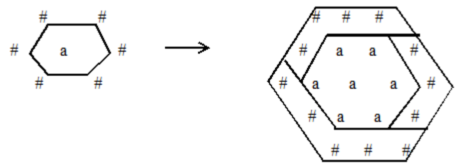


Figure. 18

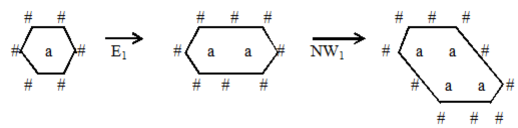


Figure. 19

2. It is shown that the picture $\hat{q}$ as a growing hexagonal picture by applying the NW, SW and E rules to get a canonical hexagonal picture. In each case there exist p_i of hexagonal pictures of size (2, 2, 2) $\subseteq \theta \wedge \hat{q}_i$ such that it generates the picture q_i. Extending q_i to $\hat{q}$ then also we get a hexagonal picture $\hat{p}$ such that $B_{2,2,2}(\hat{p}) \subseteq \theta \wedge \hat{q} = \pi(\hat{p})$. Combining all the rules and applying induction to the hexagonal picture we obtain $\hat{q} \in$ L(HRTS), which completes our proof.

Proposition 1
The hexagonal picture languages L consisting of one letter alphabet hexagonal pictures of dimension (n, n, n!) is generated by HTRS systems.
Proof
Let T = (P, R, Σ, Δ) be a HTRS system generating a language L where Σ is a finite set of alphabet and Δ contains # a bordered symbol, R is the set of rules and P is the finite set of hexagonal pictures without border versions of size (l, m, n) with n $\leq$ 3.The set of rules are defined by using dominos in an arrowhead is shown in Figure 22.

The above rule sequences can be applied to the hexagonal picture. Each rule can be iterated a number of times for obtaining the hexagonal picture. To show that a hexagonal picture language L of dimension (n, n, n!) is generated by Hexagonal tiling rule system that is L $\subseteq$ L(HTRS) we use induction for n $\geq$ 1. Since the hexagonal pictures of L having size $n \leq 3$ are generated by $\Sigma, \Sigma \subseteq$ L(HTRS). Given a hexagonal picture of size (n, n, n!) we can generate a hexagonal picture of size (n+1, n+1, (n+1)!) by using the rules mentioned above. Here we use three cases. In all the three cases let the hexagonal picture be of size (l, m, n) to satisfy the condition (n, n, n!). Let the three cases be (l = n, m = n, n = n!), (l = n!, m = n, n = n) and (l = n, m = n!, n

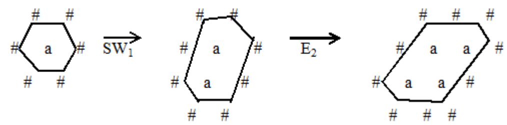


Figure. 20

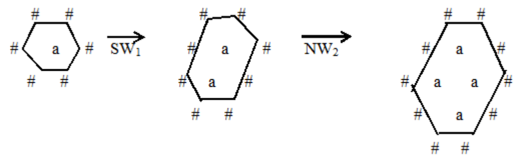


Figure. 21

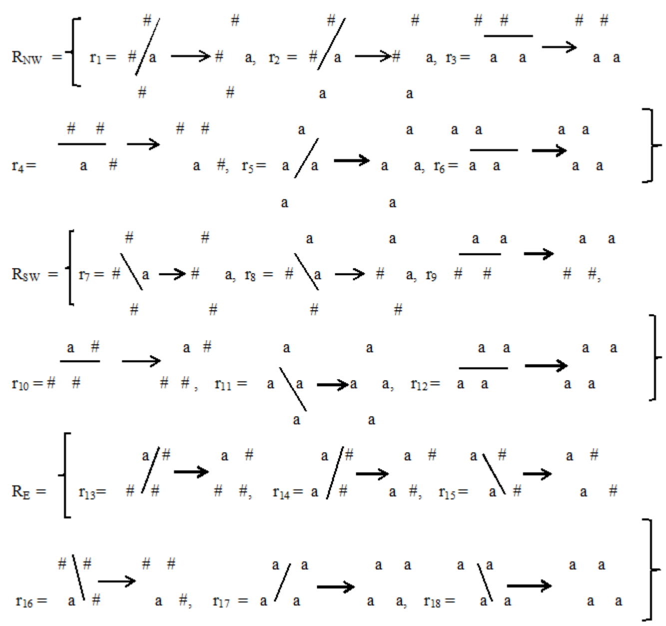


Figure. 22

= n) respectively. For convenience we take n = 2, then the hexagonal picture is of size (2, 2, 2) is used to generate a hexagonal picture of sizes (3, 3, 6), (6, 3, 3) and (3, 6, 3) by using the rules in Figure 22.
Case:1

Let P be a hexagonal picture of size (2, 2, 2!). We derive a hexagonal picture of size (3, 3, 3!) that is a hexagonal picture of size (3, 3, 6) by using the rules as follows. The sequence S of rules given by $S_1 = r_1 r_2 (r_3)_5, r_3$ is repeated 5 times, we get the NW- arrowhead of the iterated hexagonal picture. In general for a hexagonal picture of size (n, n, n!), applying the rule $r_1 r_2 (r_3)_{n!-1}$ times generates $\hat{p_1}$. Now the picture $\hat{p_1}$ is again applied the rule sequence $S_2 = r_5 (r_6)_5$ to get the picture $\hat{p_2}$. That is we get the inside NW-arrowhead of the picture $\hat{p}$. To get the remaining SW- arrowhead we use the rule sequence $S_3 = r_7 r_8 (r_9)_5$ resulting in picture $\hat{p_3}$, the inside SW- arrowhead of the picture $\hat{p}$ is obtained by using the rule sequencev $S_4 = r_{11}(r_{12})_5$. The derivation sequence d_1 is given by $d_1 = S_1 \longrightarrow S_2 \longrightarrow S_3 \longrightarrow S_4$, where

$S_1 = r_1r_2(r_3)_5, r_3, S_2 = r_5(r_6)_5, S_3 = r_7r_8(r_9)_5, S_4 = r_{11}(r_{12})_5$.
Case: 2

Here we choose l = 2, m = 2!, n = 2, to derive a hexagonal picture of size (3, 6, 3). We start from SW- rule sequence. Let $S_1 = (r_7)_5r_9r_{10}$. By applying S_1 we get the bottom SW-arrowhead say of the picture $\hat{p_1}$. Now applying the rule $S_2 = (r_{11})_5r_{12}$ we get the inside SW-arrowhead $\hat{p_2}$, again applying $S_3 = r_{13}(r_{14})_5r_{15}$ we get the hexagonal picture $\hat{p_3}$. To get $\hat{p_4}$ we apply $S_4 = r_{17}(r_{18})_5$. The derivation sequence d_2 is given by $d_2 = S_1 \longrightarrow S_2 \longrightarrow S_3 \longrightarrow S_4$ where $S_1 = (r_7)_5r_9r_{10}, S_2 = (r_{11})_5r_{12}, S_3 = r_{13}(r_{14})_5r_{15}, S_4 = r_{17}(r_{18})_5$.
Case:3

The hexagonal picture be of size (2!, 2, 2). The hexagonal picture to be derived is of size (6, 3, 3). Here we start from E arrowhead direction with E-rule sequence. As above mentioned, here the rules are $S_1 = r_{13}(r_{13})_5r15, S_2 = r_4r_3(r_2)_5S_3 = r_{15}(r_{16})_5S_4 = r_6(r_5)_5$. Applying the above rules sequentially the derivation sequence d_3 is given by
$d_3 = S_1 \longrightarrow S_2 \longrightarrow S_3 \longrightarrow S_4$
By taking d as d_1 followed by d_2 followed by d_3 we get the whole hexagonal picture $\hat{p}$. Thus the language generated by hexagonal tiling rule system is contained in the language L. That is L$\subseteq$ L(HTRS). Hence the proof.

5. CONCLUSION

Hexagonal tiling rule system provides a new formalism for defining hexagonal picture languages by assembling hexagonal tiles. Hexagonal pictures of the hexagonal picture languages are generated iteratively by applying the rules which makes the picture grow to the size (l, m, n). The rules are apllied by locating the context site and replacement site. The recognizability of hexagonal tiling system is proved earlier .The recognizability regarding the new hexagonal tiling rule system can be considered.

REFERENCES

H. Geetha, D. G. T. and Kalyani, T. 2011. Hexagonal array and its special subarrays. *Journal of Computer and Mathematical Sciences Vol.2,* No.6.

K. S. Dersanambika, K. Krithivasan, C. M.-V. and Subramanian, K. G. 2004. Hexagonal pattern languages. *Lecture Notes on Computer Science Vol.3322*, pp.52 64.

K. S. Dersanambika, K. Krithivasan, C. M.-V. and Subramanian, K. G. 2005. Local and recognizable hexagonal picture languages. *International Journal of pattern recognition and Artificial Intelligence Vol.19,* No.7.

Latteurx, M. and Simplot, D. 1997. Recognizable picture languages and domino tiling. *Theoretical Computer Science Vol.178*, pp.275 283.

Paola Bonizzoni, Claudio Ferretti, A. R. S. M. and Mauri, G. 2009. Picture languages generated by assembling tiles. *Lecture Notes on Computer Science Vol.5457*, pp.224 235.

Restivo, D. G. A. and Rozenberg, G. 1996. Two dimensional finite state recognizability. *Fundamenta Informatica Vol.25*, pp.399 422.

Restivo, D. G. A. and Rozenberg, G. 1997. Two dimensional languages. *Handbook of Formal Language, Springer Verlag Vol.III*, pp.215 268.

Siromoney, G. and Siromoney, R. 1976. Hexagonal arrays and rectangular blocks. *Computer Graphics and Image Processing 5*, 353 – 381.

Dr. Anitha.P is an Associate Professor under Collegiate Education Department, Government of Kerala, presently working at Baby John Memorial Government College, Chavara, Kollanm District. She has 6 published research papers in international journals/conferences. Anitha pursued her graduation and post graduation from the University of Kerala.She has done her PhD from Mahatma Gandhi University. Her research interests is in Automata theory.She has attended and presented papers in many interenational/national seminars.